Reducing, we have,

$$x^3+3x^2-3=0 \text{ .... } (2).$$

By Horner's method, we have from equation (2), $x=0.879385+$. Therefore

$$\frac{1}{x+3}=\frac{1}{3.839385};\quad \frac{1}{x+2}=\frac{1}{2.879385};\text{ and } \frac{1}{x+1}=\frac{1}{1.879385}.$$

It is evident that C is the best clerk and was given the 93% on the efficiency record. The records should be inversely proportional to the time expended for equivalent work. In order to compare C and B, and C and A, we have

$$x+2 : x+1=93\% : B\text{'s mark};$$
$$x+3 : x+1=93\% : A\text{'s mark};$$

and therefore,

$$2.879385 : 1.879385=93\% : 60.70\%=B\text{'s mark; and}$$
$$3.879385 : 1.879385=93\% : 45.05\%=A\text{'s mark}.$$

Thus, if C were given on the efficiency record 93%, A should be given 45.05%, and B should be given 60.70%.

Also solved by G. B. M. Zerr, S. A. Corey, G. W. Greenwood, F. D. Whitlock, R. D. Carmichael, A. H. Holmes, and J. Scheffer.

224. Proposed by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

Show that, if none of the quantities x, y, z is zero, the result of eliminating them from

$$(x+y)(x+z)=bcyz \text{ .... } (1),$$
$$(y+z)(y+x)=cazx \text{ .... } (2),$$
$$(z+x)(z+y)=abxy \text{ .... } (3),$$

is $\begin{vmatrix} \pm a, & 1, & 1 \\ 1, & \pm b, & 1 \\ 1, & 1, & \pm c \end{vmatrix}=0.$ [Oxford, 1896.]

Solution by C. H. MILLER, West Point, N. Y., and the PROPOSER.

By multiplying the second equation by the third, dividing by the first, and transposing, we obtain

$$\pm ax+g+z=0.$$

From this, and two similar equations, we get the required elimininant.

Also solved by J. B. Faught, G. B. M. Zerr, R. D. Carmichael, J. Scheffer, and J. O. Mahoney.

225. Proposed by H. M. ARMSTRONG, Cooch's Bridge, Delaware.

If $\alpha=ax+cy+bz$ .... (1), $\beta=cx+by+az$ .... (2), $\gamma=bx+ay+cz$ .... (3), show that $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma=(a^3+b^2+c^3-3abc)(x^3+y^3+z^3-3xyz)$.

Solution by the PROPOSER.

The required result follows directly from the equality,